Space–Time Turbo Trellis Codes for Two, Three, and Four Transmit Antennas

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Abstract—New space–time turbo trellis codes (ST turbo TCs) with 4-phase-shift keying (PSK) and 8-PSK for two, three, and four transmit antennas in slow and fast fading channels are proposed in this paper. The component codes of the space–time turbo schemes are constructed by choosing the feedforward coefficients to maximize the minimum squared Euclidean distance and the feedback coefficients to minimize the iterative decoding threshold. The performance of the proposed ST turbo TCs with various memory orders, transmit antennas, and interleaver structures is evaluated by simulation. It is shown that the new codes achieve better performance than previously designed codes. The impact of antenna correlation and imperfect channel estimation on the code performance is also discussed.

Index Terms—Diversity, fading channels, space–time (ST) coding, trellis codes, turbo coding.

I. INTRODUCTION

Turbo codes with iterative decoding are known to provide performance close to the Shannon capacity with large interleaver size [1]. Recently, space–time (ST) coding techniques [2] have been proposed to achieve both diversity and coding gains in multiple-input–multiple-output (MIMO) fading channels by joint design of transmit diversity and error-control coding.

In order to further improve the error performance of ST codes in MIMO fading channels, it is natural to combine ST and turbo coding techniques. Various ST turbo coding schemes [3]–[12] have been proposed in the last few years. In [7], ST trellis codes (STTCs) were first converted into recursive structure and the parallel concatenated encoder was proposed. There is no puncturing of the coded symbols in this scheme and its data rate is reduced as compared with the component codes. Recently, bandwidth efficient ST turbo trellis coding (ST Turbo TC) schemes were developed [9]–[12]. The ST turbo TCs consist of two parallel concatenated recursive STTCs. The coded sequences are constructed by using symbol interleaving and alternate parity symbol puncturing [9], [11], [12] or using bit interleaving and information puncturing [10]. The iterative decoding algorithm is employed.

In all these turbo schemes, recursive STTCs are used as component codes to benefit from interleaving gain and iterative decoding. However, there is no systematic way to construct the recursive STTCs. In [10] and [12], the recursive component codes were converted from the feedforward STTCs, which were constructed by maximizing the diversity and coding gains or the minimum squared Euclidean distance based on the code design criteria in [2] and [14], respectively. However, the feedback coefficients were chosen to be a particular value, which might not be optimized. As we will show in this paper, the feedback coefficients of the recursive STTCs do affect the turbo code performance. In particular, the feedback coefficients significantly affect the iterative decoding convergence behavior [19]–[25]. The convergence of iterative decoder can be visualized as the exchange of extrinsic information between the constituent decoders by using the extrinsic information transfer (EXIT) chart [23]. It is shown that the iterative decoder for ST turbo TCs with the same feedforward coefficients, but different feedback coefficients, converges at different decoding threshold, which is defined as the minimum signal-to-noise ratio (SNR) value, above which the iterative decoder converges and, consequently, error rate goes to zero as the number of iterations increases.

In this paper, new ST turbo TCs for two, three, and four transmit antennas are proposed by choosing the feedforward coefficients to maximize the minimum squared Euclidean distance and the feedback coefficients to minimize the iterative decoding threshold. The performance of the proposed ST turbo TCs with various memory orders, transmit antennas, and interleaver structures in MIMO fading channels is evaluated by simulation. Furthermore, performance comparison of various ST turbo TCs is carried out and it is shown that the proposed ST turbo TCs outperform the codes in [9] and [12]. Finally, the impact of antenna correlation and imperfect channel estimation on the code performance is discussed.

This paper is organized as follows. Section II introduces the system model. Section III introduces feedforward STTCs and shows how to convert feedforward STTCs into equivalent recursive codes. In Section IV, the encoder structures for ST turbo TCs and the iterative decoding with symbol or bit interleaver are presented. Section V explains the code design criteria and decoder convergence analysis for ST turbo TCs. The 4- and 8-PSK ST turbo TCs for two, three, and four transmit antennas in MIMO fading channels are proposed. Section VI presents the simulation results and, finally, a conclusion is drawn in Section VII.
II. SYSTEM MODEL

In this paper, we consider an M-PSK ST coding system with \( n_T \) transmit and \( n_R \) receive antennas. At time \( t \), the encoded M-PSK symbols \( x^t_1, x^t_2, \ldots, x^{n_T}_t \) are simultaneously transmitted over \( n_T \) antennas, where \( x^t_i, i \in \{1,2,\ldots,n_T\} \), is transmitted from antenna \( i \).

At the receiver, the signal at each of the \( n_R \) antennas is a superposition of \( n_T \) transmitted symbols scaled by channel fading and polluted by additive white Gaussian noise (AWGN). At any time \( t \), the received signal \( r^j_t \) at receive antenna \( j, j \in \{1,2,\ldots,n_R\} \), is given by

\[
r^j_t = \sqrt{E_s} \sum_{i=1}^{n_T} h^i_{k,j}(t) x^i_t + n^j_t
\]

(1)

where \( h^i_{k,j}(t) \) is the fading coefficient from transmit antenna \( i \) to receive antenna \( j \) at time \( t \). \( E_s \) is the energy per symbol at each transmit antenna. The noise component at receive antenna \( j \) at time \( t \), denoted by \( n^j_t \), is modeled as an independent sample of the zero-mean complex Gaussian random variable with a noise spectral density of \( N_0 \).

In this paper, both slow and fast fading channels will be considered. For slow fading channels, it is assumed that the fading coefficients are constant during a frame and vary from one frame to another independently. For fast fading channels, the fading coefficients are constant within each symbol period and vary from one symbol to another independently. In both fading types, the fading coefficients are modeled as independent zero-mean complex Gaussian random variables with variance 1/2 per dimension.

III. STTCs

A. Feedforward STTC

A feedforward M-PSK STTC encoder with memory order \( v \) and \( n_T \) transmit antennas is shown in Fig. 1. This STTC can achieve a bandwidth efficiency of \( m = \log_2 M \) b/s/Hz. The M-PSK STTC encoder consists of an \( m \)-branch feedforward shift register with total memory order \( v \). At time \( t \), \( m \) binary inputs \( r^k_t, k \in \{1,2,\ldots,m\} \), are fed into the \( m \) branches. The memory order of the \( k \)th branch \( v_k \) is given by

\[
v_k = \left\lfloor \frac{v+k-1}{\log_2 M} \right\rfloor
\]

(2)

where \( \lfloor x \rfloor \) denotes the maximum integer not larger than \( x \).

The \( m \) streams of input bits are simultaneously passed through their respective shift register branches and multiplied by the coefficient vectors

\[
g^1 = [(g^1_{0,1}, g^1_{0,2}, \ldots, g^1_{0,n_T}), \ldots, (g^1_{n_1,1}, g^1_{n_1,2}, \ldots, g^1_{n_1,n_T})]
\]

\[
g^2 = [(g^2_{0,1}, g^2_{0,2}, \ldots, g^2_{0,n_T}), \ldots, (g^2_{n_2,1}, g^2_{n_2,2}, \ldots, g^2_{n_2,n_T})]
\]

\[
\vdots
\]

\[
g^m = [(g^m_{0,1}, g^m_{0,2}, \ldots, g^m_{0,n_T}), \ldots, (g^m_{n_m,1}, g^m_{n_m,2}, \ldots, g^m_{n_m,n_T})]
\]

(3)

where \( g^t_{j_k,i}, k \in \{1,2,\ldots,m\}, j_k \in \{0,1,2,\ldots,v_k\}, i \in \{1,2,\ldots,n_T\} \), is an element of the \( M \)-PSK constellation set.

Fig. 1. Feedforward M-PSK STTC encoder with \( n_T \) transmit antennas.
The encoder output at time $t$ for transmit antenna $i$, denoted by $x^i_t$, can be computed as

$$x^i_t = \sum_{k=1}^{m} \sum_{j_k=0}^{\nu_i} g_{ij}^k q_{ij}^{k-j_k} \mod M, \quad i \in \{1, 2, \ldots, n_T\}. \quad (4)$$

These outputs are elements of an $M$-PSK signal set. Modulated ST symbol transmitted at time $t$ is given by

$$x_t = (x^1_t, x^2_t, \ldots, x^{n_T}_t)^T \quad (5)$$

where $T$ denotes transpose. The STTC encoder can also be described in generator polynomial format. The binary input stream $c^k$ can be represented as

$$c^k(D) = c_0^k + c_1^k D + c_2^k D^2 + \cdots + c_t^k D^t + \cdots \quad (6)$$

where $c_t^k, t \in \{0, 1, 2, \ldots\}, k \in \{1, 2, \ldots, m\}$, are binary symbols 0 and 1 and $D$ represents a unit delay operator. The feedforward generator polynomial for the transmit antenna $i$, where $i \in \{1, 2, \ldots, n_T\}$, can be represented as

$$G^i_t(D) = g_{0i}^k + g_{1i}^k D + \cdots + g_{ti}^k D^t. \quad (7)$$

The coded symbol sequence transmitted from antenna $i$ is given by

$$x^i(D) = \sum_{k=1}^{m} c^k(D) G^i_t(D) \mod M. \quad (8)$$

The relationship in (8) can be written in the form

$$x^i(D) = \left[ c^1(D) \ldots c^m(D) \right] \mod M \quad (9)$$

where

$$G_i(D) = \begin{bmatrix} G^1_i(D) \\ \vdots \\ G^m_i(D) \end{bmatrix} \quad (10)$$

is the feedforward generator matrix for antenna $i$.

A systematic 4-PSK STTC can be obtained by setting

$$G_1(D) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad (11)$$

which means that the output of the first antenna is obtained by directly mapping the input sequences into a 4-PSK sequence.

**B. Recursive STTC**

The feedforward generator matrix $G_i(D)$ for antenna $i$ in (10) can be converted into an equivalent recursive matrix by dividing it by a binary polynomial $q^k(D)$ of a degree equal to
or less than \( v_k \). The generator polynomial for antenna \( i \) of a recursive code can be represented as

\[
G_i(D) = \begin{bmatrix}
G_i^1(D) \\
G_i^2(D) \\
\vdots \\
G_i^n(D)
\end{bmatrix}
\]

(12)

where

\[
q^k(D) = 1 + q_i^k D + \cdots + q_{v_k}^k D^{v_k}
\]

(13)

and \( q_i^k \in \{0, 1\} \) are binary coefficients.

A diagram of recursive STTC encoder with \( n_T \) transmit antennas is shown in Fig. 2. The output \( x_t^i \), where \( i \in \{1, 2, \ldots, n_T\} \), can be expressed algebraically as

\[
x_t^i = \sum_{j=0}^{v_1} 2^{-j} g^1_{t-j,i} + \sum_{j=2}^{v_2} 2^{-j} g^2_{t-j,i} + \cdots + \sum_{j_m=0}^{v_m} 2^{-j_m} g^m_{t-j_m,i} \mod M
\]

(14)

where \( g^k_{t,i} \), \( k \in \{1, 2, \ldots, m\} \), is the feedforward coefficient defined as (3). The variable \( c^k_t \) is defined as

\[
c^k_t = c_t + \sum_{j_k=1}^{v_k} q^k_{t-j_k} \mod 2
\]

(15)

where \( q^k_{t-j_k} \) is a feedback coefficient defined as in (13).

Recursive STTCs have the same performance as their corresponding nonrecursive STTCs if the same sets of feedforward coefficients are used. Recursive STTCs are used as component codes in ST turbo schemes in order to benefit from interleaving gain and iterative decoding [27].

IV. ST TURBO TCs

Fig. 3 shows the encoder structure of a ST turbo TC with \( n_T \) transmit antennas. It consists of two recursive STTC encoders, one in the upper and the other in the lower branch, linked by a symbol interleaver [12]. Each encoder operates on a message block of \( L \) groups of \( m \) information bits, where \( L \) is the interleaver size. The message sequence \( c \) is given by \( c = (c_1, c_2, \ldots, c_L) \), where \( c_r \) is a group of \( m \) information bits at time \( t \), given by \( c_r = (c^1_r, c^2_r, \ldots, c^m_r) \).

The upper recursive STTC encoder maps the input sequence into \( n_T \) streams of \( L M \)-PSK symbols, \( x^1_1, x^2_2, \ldots, x^m_{n_T} \), where \( x^i_j = (x^1_{i,1}, x^2_{i,2}, \ldots, x^m_{i,L}) \), \( i \in \{1, 2, \ldots, n_T\} \). Prior to encoding by the lower encoder, the information symbols are interleaved by an odd–even symbol interleaver, which maps even symbol positions to even symbol positions and odd ones to odd ones. The symbol interleaver operates on symbols of \( m \) bits instead of on a single bit.

The lower encoder also produces \( n_T \) streams of \( L M \)-PSK symbols. Each stream is deinterleaved before puncturing and multiplexing. The deinterleaver ensures that the \( m \) information bits, which determine the encoded \( m \) binary digits of both the upper and lower encoder at a given time instant, are identical [26].

The symbol streams generated by upper and lower encoders are alternately punctured so that, at a given symbol interval, the output from only one encoder is connected to the \( n_T \) antennas. For example, in a system with two transmit antennas and 4-PSK modulation, if the outputs from the first and second encoder in the first three symbol intervals are defined as \( x_{1,1}, x_{1,1}, x_{1,2}, x_{1,2}, x_{1,3}, x_{1,3} \) and \( x_{2,1}, x_{2,1}, x_{2,2}, x_{2,2}, x_{2,3}, x_{2,3} \), respectively, the punctured transmitted sequence is \( x_{1,1}, x_{1,1}, x_{1,2}, x_{1,2}, x_{1,3}, x_{1,3} \). The restriction of odd–even interleaver is introduced to ensure that there are no erasures in successive symbol intervals in component decoders for the punctured scheme. The spectral efficiency of this scheme is \( m \) b/s/Hz.

Interleaving can be done on bit, rather than symbol, streams [10]. The punctured scheme imposes a restriction on the bit interleaver that is similar to the one in the symbol interleaver. In this case, interleaving is performed by two bit-wise interleavers of size \( L \) for 4-PSK modulation. One interleaver operates on odd input symbol positions and the other operates on even symbol positions. For example, for 4-PSK modulation, if the input sequence is \( c^1, c^2, c^3, c^4, c^5, c^6, c^7, c^8, c^9, c^{10}, c^{11}, c^{12} \), the first interleaver will scramble positions \( c^1, c^2, c^3, c^4, c^5, c^6, c^7, c^8, c^9, c^{10} \), corresponding to the odd 4-PSK symbols, while the second interleaver will scramble positions \( c^1, c^2, c^3, c^4, c^5, c^6, c^7, c^8, c^9, c^{10} \), corresponding to the even 4-PSK symbols. In general, the encoder output can be only multiplexed, without being punctured, giving the spectral efficiency of \( m/2 \) b/s/Hz.

For ST turbo TCs with symbol interleaving, a symbol-by-symbol maximum a posteriori (MAP) probability algorithm is employed in the iterative decoding at the receiver. This decoding process is similar to the binary turbo code, except that the symbol probability is used as the extrinsic information rather than the bit probability.

For ST turbo TCs with bit interleaving, the decoding process can be carried out by converting the joint systematic and extrinsic information computed for a symbol to a bit level, since
the exchange of the information between the decoders is on a bit level. After interleaving or deinterleaving operation, the priori probabilities need to be converted to a symbol level, since they will be used in the branch-transition probability calculations [26].

V. CODE DESIGN

The performance analysis and code design for ST codes in slow and fast fading channels have been presented in [2]. Assume that a codeword \( \mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_L) \) is transmitted, where \( L \) is the codeword length and \( \mathbf{x}_t = (x_{t1}^1, x_{t2}^1, \ldots, x_{tN_T}^1)^T \) is a ST symbol at time \( t \). A maximum-likelihood receiver might decide erroneously in favor of another codeword \( \hat{\mathbf{x}} = (\hat{x}_1^1, \hat{x}_2^1, \ldots, \hat{x}_L^1) \), where \( \hat{x}_t = (\hat{x}_{t1}^1, \hat{x}_{t2}^1, \ldots, \hat{x}_{tN_T}^1)^T \). Let \( r \) denote the rank of the \( N_T \times L \) codeword difference matrix

\[
B(\mathbf{x}, \hat{\mathbf{x}}) = \begin{bmatrix}
    x_{11}^1 - \hat{x}_{11}^1 & x_{12}^1 - \hat{x}_{12}^1 & \cdots & x_{1L}^1 - \hat{x}_{1L}^1 \\
    x_{21}^2 - \hat{x}_{21}^2 & x_{22}^2 - \hat{x}_{22}^2 & \cdots & x_{2L}^2 - \hat{x}_{2L}^2 \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{N_T1}^{N_T} - \hat{x}_{N_T1}^{N_T} & x_{N_T2}^{N_T} - \hat{x}_{N_T2}^{N_T} & \cdots & x_{N_TL}^{N_T} - \hat{x}_{N_TL}^{N_T}
\end{bmatrix}
\]

(16)

and \( \lambda_i \) be the eigenvalues of the codeword distance matrix \( \mathbf{A} = \mathbf{B} \mathbf{B}^* \), where * denotes the transpose conjugate. The pairwise error probability (PWE) is given by [2]

\[
P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq \left( \prod_{i=1}^{r} \lambda_i \right)^{-n_R} \left( \frac{E_b}{4N_0} \right)^{-n_R}
\]

(17)

for slow fading channels and

\[
P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq \left( \prod_{t \in \rho(\mathbf{x}, \hat{\mathbf{x}})} |x_t - \hat{x}_t|^2 \right)^{-n_R} \left( \frac{E_b}{4N_0} \right)^{-\delta_H n_R}
\]

for fast fading channels, where \( \delta_H \) is the minimum symbol Hamming distance between \( \mathbf{x} \) and \( \hat{\mathbf{x}} \). \( \rho(\mathbf{x}, \hat{\mathbf{x}}) \) denotes the set of time instances \( t \in \{1, 2, \ldots, L\} \) such that \( |x_t - \hat{x}_t|^2 \neq 0 \). It is shown that the code error performance is determined by the minimum rank \( r \) of the codeword distance matrix over all pairs of distinct codewords and its minimum product of nonzero eigenvalues of the distance matrix \( \prod_{i=1}^{r} \lambda_i \) in slow fading channels or the minimum symbol-wise Hamming distance \( \delta_H \) and the product distance over all pairs of distinct codewords \( \prod_{t \in \rho(\mathbf{x}, \hat{\mathbf{x}})} |x_t - \hat{x}_t|^2 \) in fast fading channels [2]. In [14], it is proposed that for systems with a large number of subchannels (for example, if \( r n_R \geq 4 \) for slow fading channels and \( \delta_H n_R \geq 4 \) for fast fading channels), a large number of diversity branches reduce the effect of fading and, consequently, drive the fading channel toward an AWGN channel. In this case, we have

\[
P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq \frac{1}{4} \exp \left( -n_R \frac{E_b}{4N_0} \sum_{i=1}^{r} \lambda_i \right)
\]

(18)

\[
= \frac{1}{4} \exp \left( -n_R \frac{E_b}{4N_0} \sum_{i=1}^{r} \sum_{j=1}^{L} |x_j - \hat{x}_j|^2 \right).
\]

Thus, the code performance is dominated by the minimum squared Euclidean distance or, equivalently, the minimum trace of the codeword distance matrix.

It is discussed in [15] and [17] that, for slow fading channels, the rank and determinant criteria apply to the system with a single receive antenna and small number of transmit antennas, while the Euclidean distance criterion applies to systems with two or more receive antennas or to systems with a large number of transmit antennas. For fast fading, it is always possible to achieve a minimum value 4 for \( \delta_H n_R \) for two or more receive
TABLE I
NEW 4-PSK ST TURBO TCS

<table>
<thead>
<tr>
<th>$n_f$</th>
<th>$v$</th>
<th>Feedforward Coefficients</th>
<th>Feedback Coefficients</th>
<th>$d_c^0$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>$[0.2,(1,2)]$</td>
<td>$[2.(3),(2,0)]$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$[(2.2),(2,1)]$</td>
<td>$[(2.0),(2,2),(0.2)]$</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$[(1.2),(3,2)]$</td>
<td>$[(2.0),(2,2),(0.2)]$</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>$[0.2,(2.3),(1,2)]$</td>
<td>$[(2.2),(2.3),(2,0)]$</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>$[0.2,(3),(3,3),(3,2)]$</td>
<td>$[(2.2),(2.2),(0,0),(2,0)]$</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$[(2.2),(2,1,2)]$</td>
<td>$[(2,3),(2,0,1)]$</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$[(2.2),(1,2,3)]$</td>
<td>$[(2,0),(1,2),(0,2,2)]$</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
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<td>$[(2,0),(1,2,2),(3,1),(2,0,0)]$</td>
<td>7</td>
<td>24</td>
</tr>
<tr>
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<td>2</td>
<td>$[(0.2),1,(2,2,2)]$</td>
<td>$[(2,3),(2),(0,1,1)]$</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
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<td>3</td>
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<td>$[(2,0),(1,2),(0,1),(2,2,3)]$</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
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<td>$[(2,0),(2,2),(0,2,0),(2,0,2,2)]$</td>
<td>3</td>
<td>32</td>
</tr>
</tbody>
</table>

Since $\text{SNR}_{1\text{out}} = \text{SNR}_{1\text{in}}$, we have

$$\text{SNR}_{2\text{out}} = G_2 \left( G_1 \left( \frac{\text{SNR}_{1\text{in}}, E_b}{N_0} \right) \right).$$  \hspace{1cm} (22)

To visualize the effect of feedback coefficients on the code performance, we consider an example in which the EXIT charts and the error performance of two 32-state 4-PSK ST turbo TCs with two transmit antennas are plotted and compared. The two codes are denoted by $C_1$ and $C_2$, respectively, and have the same feedback coefficients $g^2 = [(0,2),(2,3),(1,2)]$ and $g^2 = [(2,2),(1,2),(2,3),(2,0)]$ but different feedback coefficients, which are $q^1 = 1 + D + D^2, q^2 = 1 + D + D^3$ for $C_1$ and $q^1 = 1 + D + D^2, q^2 = 1 + D + D^2 + D^3$ for $C_2$. In the following performance evaluations, two receive antennas are assumed. The interleaver size is 1024 symbols, which is the same as the frame size.

Fig. 5 depicts the decoder convergence of codes $C_1$ and $C_2$ at $E_b/N_0 = 1$ dB. The EXIT charts in Fig. 5 show the progress of the decoder's iterations. The left chart for code $C_1$ shows that there is a large decoding tunnel between $G_1$ and $G_1^{-1}$ curves at $E_b/N_0$ of 1 dB, through which the iterative decoding progresses, while the right chart for code $C_2$ shows that the $G_1$ and $G_2^{-1}$ curves almost touch each other at the same $E_b/N_0$. For code $C_2$, this means that there is less improvement in SNR for extrinsic information in the iteration process and the performance improves at a slower pace with an increasing number of iterations compared to code $C_1$. Thus, the code $C_1$ has a smaller iterative decoding threshold and better iterative decoding convergence than code $C_2$, where the iterative decoding threshold is defined as the smallest $E_b/N_0$ value beyond which an iterative
decoder converges and the error rate goes to zero as the number of iterations increases.

In order to justify the assessment of decoder convergence shown in Fig. 5, Fig. 6 depicts the code performance for codes $C_1$ and $C_2$ in slow fading channels. The figure shows that code $C_1$ outperforms code $C_2$ by 0.8 dB at a frame error rate (FER) of $10^{-3}$ with ten iterations, which verifies that the code performance is affected by the variations of the feedback polynomials.

Fig. 7. Performance of four-, eight-, and 16-state 4-PSK ST turbo TCs with symbol interleaving, 2 b/s/Hz, slow fading.

Fig. 8. Performance of eight- and 16-state 8-PSK ST turbo TCs with symbol interleaving, 3 b/s/Hz, slow fading.
It also indicates that we can use the EXIT charts and iterative decoding threshold to choose good feedback polynomials.

Based on the previous discussions, we formulate the code design criteria in both slow and fast fading channels as follows:

1) choosing feedforward coefficients based on the Euclidean distance criterion provided that \( \tau_{H} \geq 4 \) for slow fading and \( \delta_{H} \geq 4 \) for fast fading;
2) choosing feedback coefficients based on iterative decoding convergency by minimizing the iterative decoding threshold in EXIT chart.

New recursive \( M \)-PSK STTCs for a given number of transmit antennas and memory order are designed by applying the criteria. The recursive STTCs are used as component codes in the ST turbo TCs. In the code design, for the given encoder structure, a set of feedforward and feedback coefficients are determined by exhaustive search. It is important to note that the STTC encoder structure cannot guarantee geometrical uniformity of the code. Therefore, the search is conducted over all possible pairs of paths in the code trellis.

Tables I and II list the proposed 4- and 8-PSK ST turbo component codes with bandwidth efficiency 2 and 3 b/s/Hz, respectively, for two, three, and four transmit antennas. The codes are described by the memory order \((v)\), feedforward coefficients \( (g^{m}) \), feedback coefficients in octal form \( (q^{m}) \), the minimum squared Euclidean distance \( (d_{PE}) \), and the minimum rank \( r \). For example, the four-state 4-PSK ST turbo TC with \( \nu_{T} = 2 \) and \( v = 2 \) in Table I has the feedforward coefficients as

\[
\begin{align*}
g^{1} &= [(g_{0,1}^{1}, g_{0,2}^{1})], \\
g^{2} &= [(g_{0,1}^{2}, g_{0,2}^{2})], \\
\end{align*}
\]

(23)

Its feedback coefficients are \( (q^{1} = 1 + D, q^{2} = 1 + D) \), which are represented in octal form as \( (q^{1} = 3, q^{2} = 3) \).

The proposed codes with two transmit antennas in Tables I and II have a full rank \( r = 2 \). For codes with three and four transmit antennas, the minimum rank \( r \) is at least 2. Considering \( \delta_{H} \geq r \) and \( \nu_{T} = 2 \), we have \( \tau_{H} \geq 4 \) and \( \delta_{H} \geq 4 \) for all codes. In this case, the Euclidean distance criterion applies in slow and fast fading channels and these codes are good for both types of channels [14].

VI. PERFORMANCE SIMULATION

The performance of the proposed ST turbo TCs with various numbers of transmit antennas and system parameters is evaluated by simulation. Ten iterations are assumed and each frame consists of 130 symbols out of each transmit antenna.

Figs. 7 and 8 plot the performance of ST turbo TC with symbol interleaver for various numbers of transmit antennas and two receive antennas in slow fading channels, for 4- and 8-PSK signal sets, respectively. Fig. 7 shows that increasing the number of transmit antennas from two to three brings a gain of about 2 dB at the FER of \( 10^{-3} \). A further increase in the number of transmit antennas from three to four results in a smaller gain of about 1 dB.

For two transmit antennas, increasing the component code memory order does not improve the FER performance, as the fading coefficients make it more difficult for an iterative decoder with a higher memory order to converge. This is similar to the case in AWGN channels, where increasing the memory order of turbo component codes from three to five cannot improve the error performance in the “waterfall” reign, due to the poor convergence behavior of a higher memory order code [28]. It is demonstrated in Fig. 7 that the performance of 16-state component code with two transmit antennas is even worse than that of the one with the four-state code, which is about 2 dB away from the outage probability. Here, the outage probability serves as an FER lower bound in slow fading channels. However, as the number of transmit antennas increases from two to four, the higher memory component codes perform better due to more independent fading coefficients and larger diversity gain. A similar behavior can be observed for 8-PSK ST turbo TCs as the number of the transmit antennas varies, as depicted in Fig. 8.

Fig. 9 shows the FER performance comparison of the ST turbo TCs scheme and STTCs [15] with two, three, and four transmit and two receive antennas in slow fading channels. For
two transmit antennas, the new turbo scheme offers a gain of about 2.8 dB over the STTCs at the FER of $10^{-3}$. As the number of the transmit antennas increases from two to three and four, the turbo schemes outperform the corresponding STTCs by about 2.9 and 2.2 dB, respectively, at the FER of $10^{-3}$.

Fig. 10 shows the FER performance comparison of the proposed four- and 16-state 4-PSK ST turbo TCs and the codes in [9] and [12] for two transmit in slow fading channels. We can see that for two receive antennas the 4-PSK codes that proposed in this paper outperform the corresponding codes in [9] and [12] by 0.3 dB at the FER of $10^{-3}$. Fig. 10 also illustrates the performance comparison for one receive antenna. It is shown that the proposed codes perform better than those in [9] and [12] by 0.5 and 0.8 dB, respectively, at the FER of $10^{-2}$. The performance of the proposed 16-state code is 2.8 dB away from the outage probability for two transmit and one receive antennas.

In fast fading channels, the performance of ST turbo TC improves as the memory order increases. This is depicted in Fig. 11 for systems with two transmit and two receive antennas and the interleaver size of 130 symbols. Comparing Figs. 7 and 11, it is obvious that the eight-state component code is the best choice if a system is required to operate over both fast and slow fading channels for two transmit antennas. The ST turbo TC with the eight-state component code performs slightly worse than the one with the four-state component code in slow fading channels and better by almost 1.5 dB at the FER of $10^{-3}$ than the system with the four-state component code in fast fading channels.

There is a possible choice of bit or symbol interleaver in this scheme. Fig. 12 depicts the code performance comparison of the four-state 4-PSK ST turbo TCs with bit and symbol interleaver in slow and fast fading channels. It is shown that the code performance for the symbol and bit interleaving is similar in slow
fading channels, while the code performance for bit interleaving has an advantage over that for symbol one in fast fading channels. This advantage is only 0.3 dB at the FER of $10^{-3}$, but the decoding complexity with bit interleaving is higher than the one with symbol interleaving [26].

Fig. 13 illustrates the performance of the ST turbo TC scheme in correlated slow fading channels. It is assumed that there is only correlation between transmit antennas. The performance of ST turbo TC is slightly degraded in slow fading channels for a correlation factor of 0.25 and 0.5. For the correlation factor of 0.75, the FER degrades by 1.8 dB relative to the case of uncorrelated antennas. The results indicate that the channel correlation between transmit antennas gives minor performance degradation when the correlation factor is less than 0.5.

Fig. 14 depicts the impact of imperfect channel estimation on the performance of ST turbo TCs. Channel estimation in the simulations was carried out by transmitting 10 orthogonal preambles in a slow fading channel. The loss of SNR at the FER of $10^{-3}$ is about 0.7 dB relative to the ideal channel estimation. The performance degradation accounts for the energy loss due to the training sequence and the error of channel estimation and indicates that the ST turbo TC is not very sensitive to channel estimation error.

VII. CONCLUSION

New ST turbo trellis codes for 4- and 8-PSK with two, three, and four transmit antennas are proposed in this paper. The new codes are constructed by maximizing the minimum squared Euclidean distance and minimizing the iterative decoding threshold. The performance of the proposed codes with various memory orders, transmit antennas, and interleaver structures is evaluated by simulation. It is shown that the proposed codes outperform the known codes. The impact of antenna correlation and imperfect channel estimation on the code performance is discussed.

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REFERENCES

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